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Short Papers

Higher Order Modes in Square Coaxial Lines

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Abstract—**The cutoff frequencies of higher order modes in square coaxial lines are presented and compared with those of circular coaxial lines having the same mean circumference. It is noted that while the characteristics of the dominant TE_{10} mode and the next TE_{11} mode in the square line differ but little from those of their circular counterparts, the same conclusions do not hold in general for the remainder of the mode spectrum.**

It is well known that a pair of independent waves having a horizontal and a vertical polarization may be supported by waveguides having either a circular or a square cross section. The same conclusion holds for circular coaxial lines as well as square coaxial lines.

While circular coaxial lines have been used extensively in the past, square coaxial lines may be preferable in some applications if a) the presence of flat rather than circular surfaces offers mechanical advantages, and b) it is desired to have an unambiguously defined plane of polarization. Moreover, it may be conjectured that in practice, at least in some instances the cross-polarization ratio, or ability to discriminate against waves having the undesired alternative polarization, may be superior for square lines.

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The higher order mode spectrum of circular coaxial lines is very well known [1]. This may be contrasted with the fact that published information pertaining to square lines is very incomplete and inadequate for most purposes. Thus a method for the determination of the lowest (TE_{10} or TE_{01}) eigenvalues of transmission lines having rectangular inner and outer conductors has been described [2] but no explicit information applicable to square lines is available. The paper by Brackelmann *et al.* [3] deals with rectangular lines comprising inner and outer conductors, the centers of which do not necessarily coincide and which include coaxial lines as a special case; from the curves, a few selected values of the cutoff frequencies of a few modes of square coaxial lines may be deduced. Tourneur [4] arrived at the higher order mode spectrum of a square coaxial line using a finite element method and a variational Rayleigh–Ritz procedure with a polynomial approximation; published information is confined to curves of the cutoff frequencies of the TE_{10}, TE_{11}, TE_{20}, and TM_{11} modes for b/a ratios ranging from 0 to 0.3. Finally, the author has independently described [5]–[7] a technique based on field matching (just as in papers by Bezlyudova and Brackelmann *et al.* [2] and [3], but differing in implementation), applicable to rectangular coaxial lines.

The same computer program which was used to arrive at the higher order mode spectrum of rectangular coaxial lines having arbitrary inner and outer conductor dimensions, has been utilized to deduce the characteristics of square coaxial lines. Calculations were performed for aspect ratios b/a ranging from 0 to 0.95 and the results extrapolated to include $b/a = 1$; it may be noted that

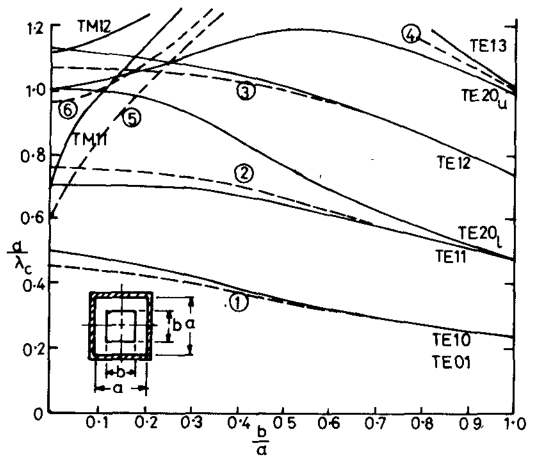

Fig. 1. Normalized cutoff frequencies of a square coaxial transmission line. For comparison, cutoff frequencies of a circular coaxial line having the same mean circumference are shown using dashed lines (curve 1—TE_{11} mode; curve 2—TE_{21} mode; curve 3—TE_{31} mode; curve 4-TE_{41} mode; curve 5—TM_{01} mode; and curve 6—TE_{01} and TM_{11} modes).

when $b/a = 1$ propagation cannot take place since the gap between the conductors vanishes.

Examination of the curves of Fig. 1 shows that, as one would expect, the TE_{10} mode is the dominant higher order mode which may propagate in addition to the TEM mode. The degeneracy of the TE_{01} and TE_{10} modes is preserved in the sense that the dependence of their cutoff wavelengths on the aspect ratio b/a is the same; the field distribution of the TE_{01} mode is shifted by 90° relative to that of the TE_{10} mode, its form being otherwise identical. It can be noted that the cutoff wavelengths of the TE_{10} and the TE_{01} modes increase monotonically as the aspect ratio b/a is increased.

The next higher order mode is the TE_{11} mode, followed by either the TM_{11} mode for $b/a < 0.1$ or the TE_{20} mode for $b/a > 0.1$. For small gaps between the inner and outer conductors, i.e., b/a approaching unity, the cutoff frequency of the TE_{20} mode is only marginally higher than that of the TE_{11} mode.

It may be noted that the degeneracy of the TE_{20} and TE_{02} modes has been removed by the presence of the inner conductor and the TE_{20} mode effectively splits up into the lower TE_{20_l} and the upper TE_{20_u} modes. This may be understood with reference to the symmetry properties of the structure [8]. A rather similar phenomenon was observed by Stalzer *et al.* [9] who investigated the mode spectrum of hollow crossed square waveguides.

Cutoff wavelengths for the TE_{10} and TE_{11} modes of a square coaxial line are shown in Table I for five aspect ratios b/a. With reference to [5], [6], these values were arrived at by progressively increasing the size of the determinant and by studying the convergence of the results obtained by setting it equal to zero. For all practical purposes, the accuracy of the results obtained with the aid of the foregoing technique is governed by the amount of computation time used (although eventually it is limited by roundoff errors). If desired, the above figures may be interpolated for design purposes and more accurate results arrived at than those obtainable by inspection of Fig. 1.

It should be noted that cutoff wavelengths of higher order modes are also needed if knowledge of the field distribution is required; this information can be deduced using the foregoing theory [5], [6], although the computational effort is considerably greater than that required to arrive at the field distribution in a circular coaxial line, for which explicit expressions are available [1].

TABLE I
CUTOFF WAVELENGTHS OF TE_{10} AND TE_{11} MODES; $a = 1$

b/a	0.1	0.3	0.5	0.7	0.9
λ_c TE10	2.044	2.351	2.793	3.268	3.755
λ_c TE11	1.415	1.438	1.540	1.706	1.900

For comparison, the characteristics of the higher order modes in circular coaxial lines [1], [10] have been shown in Fig. 1 using dashed lines. The outer and inner radii of the circular line are assumed to be equal to $2b/\pi$ and $2a/\pi$, respectively, thus making the mean circumferences of the corresponding square coaxial and circular coaxial lines the same and equal to $2(a+b)$ in both cases.

It is clear that the characteristics of the dominant TE_{10} mode in the square line differ but little from those of the TE_{11} mode in the circular coaxial structure (using the nomenclature of Marcuvitz and shown in curve 1). The same conclusions hold for the TE_{11} mode in the square line and the TE_{21} mode in the circular structure (curve 2) as well as the TE_{21} mode in the square line and the TE_{31} mode in the circular structure (curve 3), respectively.

On the other hand, there is little, if any, correspondence between various TM-mode characteristics of the two lines, and there is no counterpart for the TE_{01} mode in the circular line (curve 6, same as for the TM_{11} mode of the circular structure) or for the upper branch of the TE_{20} mode in the square line.

To summarize, at least for low values of the index n, the characteristics of the $TE_{n,1}$ $(n = 0,1,2,\cdots)$ modes in the square coaxial lines can be estimated by reference to the respective $TE_{n+1,1}$ modes of a circular coaxial line having the same mean circumference.

In general, however, if the mode spectrum as well as the field distribution of the square coaxial line are required, a direct study of the latter is called for.

ACKNOWLEDGMENT

The author would like to express his appreciation to the reviewers for drawing his attention to the work by Bezlyudova [2] and for their many helpful comments.

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The Resonant Frequency of Rectangular Interdigital Filter Elements

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***Abstract*—A procedure is given for the computation of the resonant frequency of loosely coupled interdigital resonators with rectangular cross section. The procedure is based on the use of Getsinger's fringing capacitance data [1]. The accuracy of the method was verified experimentally and found to be approximately 1 percent for a 2-percent bandwidth interdigital linear-phase filter.**

I. Introduction

Certain microwave structures, such as interdigital filters, are constructed using an array of parallel coupled rectangular cross-sectional resonators [1]. The side view of an interdigital resonator is shown in Fig. 1 and a plan view in Fig. 2. The geometry of the resonator end is shown in Fig. 3. The resonator has width w, thickness t, and length l'. It is symmetrically enclosed in a cavity of length l, formed by two parallel plates with ground plane spacing b. The cavity is filled with a homogeneous dielectric of relative permittivity ϵ_r. One end of the resonator is short circuited by the vertical wall of the cavity, while the open end is separated from the other vertical wall by a gap of length g.

It is assumed that only the TEM mode propagates, and that the interdigital resonator can be represented by the equivalent circuit of Fig. 4 where Z_0 is the characteristic impedance of the rectangular cross-sectional resonator at the center frequency, and C_g is a lumped capacitance due to the gap. Z_0 is determined by the cross-sectional dimensions, w, t, and b, and the spacing of adjacent resonators. In practice, once w, t, and b have been selected, the problem in resonator design is to find the gap length g, which yields the correct gap capacitance C_g, for a specified resonant frequency f_0.

The problem of computing the gap capacitance has been addressed by Nicholson [2] and Khandelwal [3]. Nicholson's procedure is for circular cross-sectional resonators. Khandelwal's more elaborate procedure is useful for general cross sections. Incidentally, Khandelwal's procedure for computing the fringing capacitance between the resonator tip and the end, top, and bottom plates [3, fig. 2] is incorrect because of the addition of $2C'_{fe}$ to $2C'_{fo}$. Getsinger's odd-mode capacitance $2C'_{fo}$ is the total fringing capacitance to ground, and includes the effect of top and bottom plates as well as the end plate [1, fig. 6(a)].

The procedure described here is applicable to loosely coupled resonators of rectangular cross section, is simple to use, and has given good results in the design of a narrow-band interdigital linear-phase filter.

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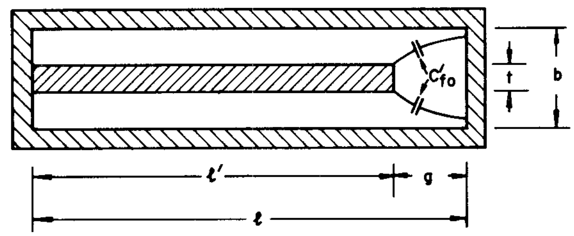

Fig. 1. Side view of an interdigital resonator.

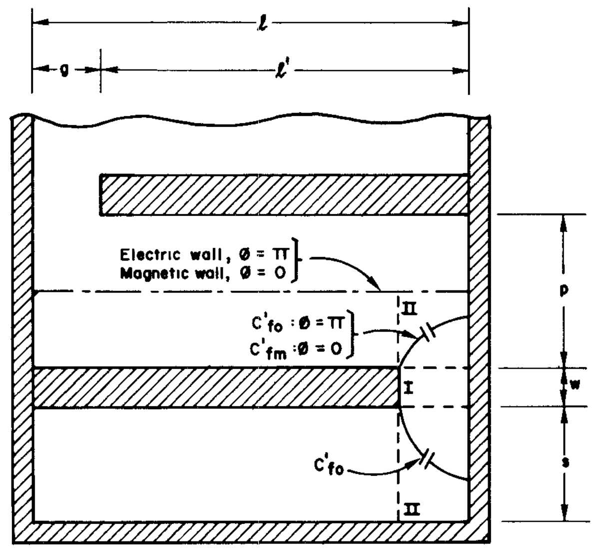

Fig. 2. Plan view of an end resonator showing boundary conditions.

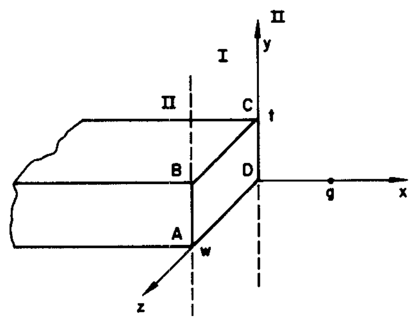

Fig. 3. Geometry of the resonator end.

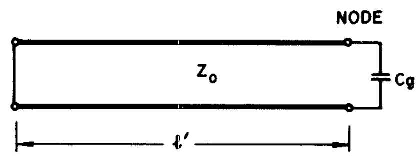

Fig. 4. Resonator equivalent circuit.

II. The Gap Capacitance

The cavity length is

$$l = \lambda_0 / (4\sqrt{\epsilon_r}) \tag{1}$$

where λ_0 is the free-space wavelength at the desired resonant frequency f_0.